

Non-linear response and event plane correlations

Derek Teaney and Li Yan¹

Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York, 11790, USA

Abstract

We apply a non-linear flow response formalism to the recently measured event plane correlations. We find that the observed event plane correlations can be understood as an average of the linear and quadratic response.

1. Introduction

The Quark-Gluon Plasma (QGP), as observed in heavy-ion collisions at RHIC and the LHC, exhibits strong collective flow [1, 2, 3, 4]. The observed flow pattern is characterized by long range correlations in the final state particle spectrum. Indeed, ATLAS recently measured event plane correlations [5], and significant two-plane and three-plane correlations were observed. In comparison to the v_n measurements, the event plane correlations provide additional insight into the origin of initial state fluctuations and additional constraints on the shear viscosity of the QGP. Although these correlations can be simulated with event-by-event hydrodynamics [6], it is important to explain the observed correlations without elaborate computer models. The aim of this paper is to explain the correlations, by taking into account the non-linear mixing of modes through quadratic order in “single-shot” hydrodynamics [7, 8].

2. Methodology

Event plane correlations can be obtained in single-shot hydrodynamics using a quadratic response formalism [9]. As detailed in [9], the response formalism uses the cumulant expansion (as opposed to a moment expansion) to characterize the initial geometry [10]. The medium response to a given cumulant is calculated at linear and quadratic order by deforming a Gaussian distribution and calculating the response coefficients with ideal and viscous hydrodynamics.

For example, the fourth order cumulant C_4 and angle Φ_4 are closely related to the fourth order eccentricity $\langle r^4 e^{i4\phi_s} \rangle$, and characterize the quadrangular deformation of energy density in the transverse plane

$$C_4 e^{i4\Phi_4} = \langle r^4 e^{i4\phi_s} \rangle - 3 \langle r^2 e^{i2\phi_s} \rangle^2.$$

Using cumulants rather than moments to characterize the initial state geometry leads to different correlations between the angles Φ_n . Fig. 1a shows the correlations between Φ_2 and Φ_4 with cumulant and moment based definitions.

¹Speaker, email: li.yan@stonybrook.edu

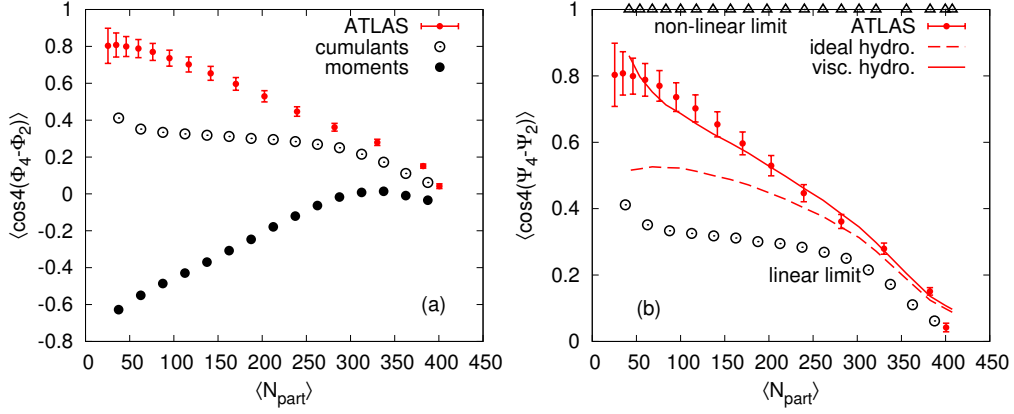


Figure 1: (a) Initial participant plane Φ_2 and Φ_4 correlation, as a function of centrality. Simulations are done with PHOBOS Glauber model with Φ_4 from both cumulants and moments definitions. The data points are extracted from ATLAS [5] for comparison. (b) Event plane Ψ_2 and Ψ_2 correlation, with predictions made by ideal and viscous hydrodynamics.

The particle spectrum is expanded in harmonics

$$\frac{dN}{d\phi_p} = \frac{N}{2\pi} \left[1 + \sum_n v_n e^{in(\phi_p - \Psi_n)} + c.c. \right] = \frac{N}{2\pi} \left[1 + \sum_n z_n e^{in\phi_p} + c.c. \right], \quad (1)$$

and each harmonic and angle, $z_n \equiv v_n e^{-in\Psi_n}$, will receive contributions from both the linear and quadratic response to a given cumulant. For v_4 for example, the dominant quadratic response is proportional to $(\epsilon_2 e^{i2\Phi_2})^2$, and the linear response is proportional to the fourth order cumulant $C_4 e^{i4\Phi_4}$

$$z_4 = v_4 e^{-i4\Psi_4} = w_4 e^{-i4\Phi_4} + w_{4(22)} e^{-i4\Phi_2}, \quad (2)$$

where w_4 and $w_{4(22)}$ indicate linear response to C_4 and the non-linear response to ϵ_2^2 .

It is straightforward then to express the event plane correlations using the complex form of harmonic flow z_n in Eq. (1), *e.g.*

$$\cos 4(\Psi_4 - \Psi_2) = \frac{\text{Re}[z_2^2 z_4^*]}{|z_2|^2 |z_4|} = \frac{\cos 4(\Phi_4 - \Phi_2) w_4 + w_{4(22)}}{|w_4 e^{-i4\Phi_4} + w_{4(22)} e^{-i4\Phi_2}|}. \quad (3)$$

Eq. (3) shows the typical structure of event plane correlations in the framework of non-linear response. In the numerator and denominator one term is determined by the linear response, w_4 , and one term is determined from quadratic response, $w_{4(22)}$. In more involved cases, where higher order non-linear corrections are also included, there are interference terms between the linear and quadratic response. The combined effects of all these linear, non-linear, and interference terms determines the behavior of the event plane correlations.

Fig. 1(b) shows that if linear response w_4 dominates the v_4 signal, the event plane correlation reduces to its corresponding initial participant plane correlation, $\langle \cos 4(\Phi_4 - \Phi_2) \rangle$. On the other hand, if the v_4 signal is dominated by the non-linear response, $w_{4(22)}$, the event planes Ψ_4 and Ψ_2 would be perfectly correlated. Referring to these two limits as the linear and non-linear limits respectively, the observed event plane correlations can be understood as an average of

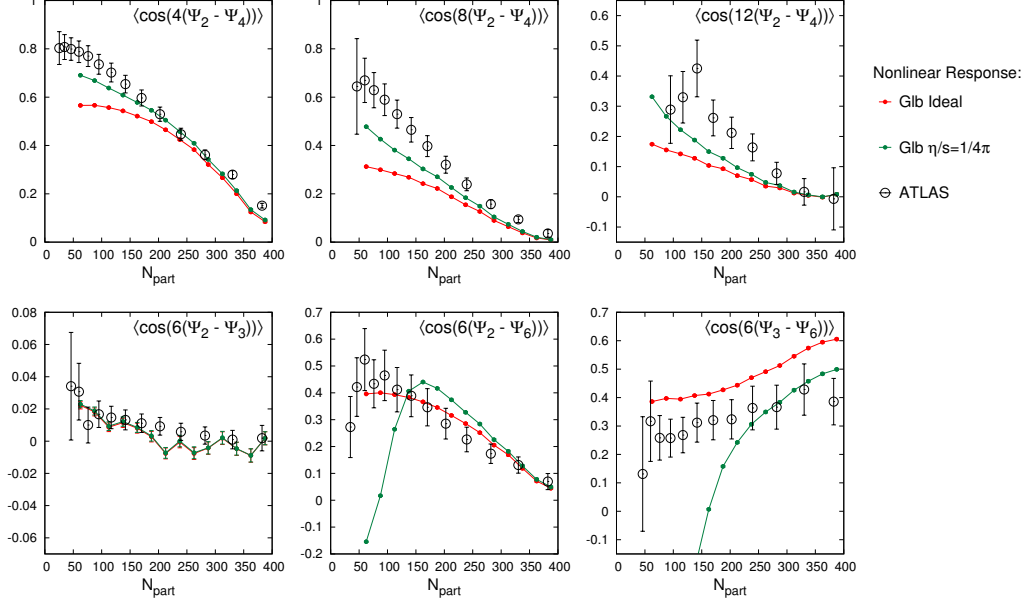


Figure 2: Two-plane correlations of event plane correlations measured at ATLAS[5].

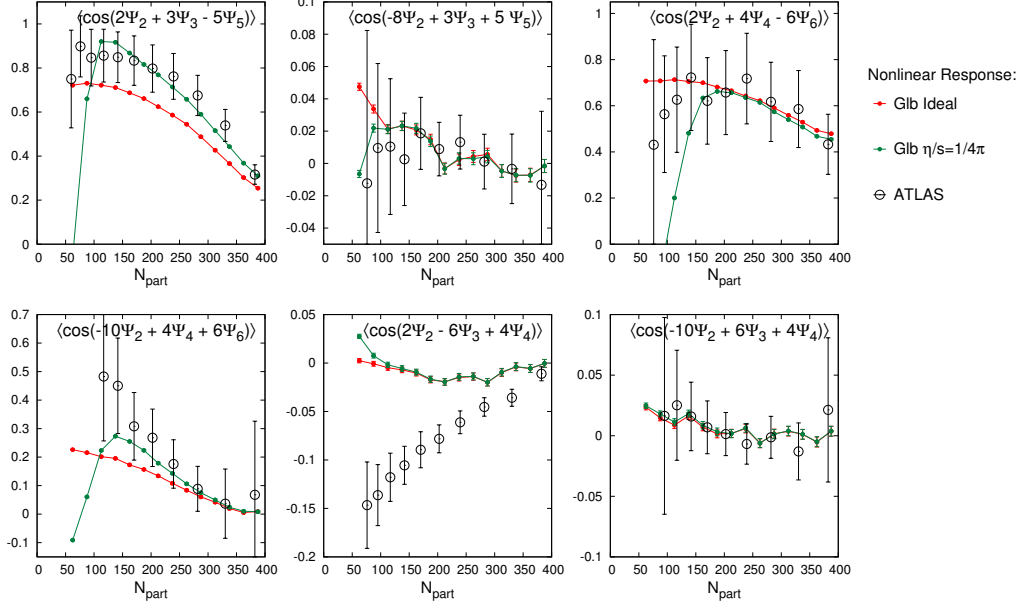


Figure 3: Three-plane correlations of event plane correlations measured at ATLAS[5]. The non-linear flow generations we considered in these calculations are: $w_{3(12)}$ for v_3 , $w_{4(22)}$ for v_4 , $w_{5(23)}$ for v_5 , and $w_{6(33)}$, $w_{6(24)}$, $w_{6(22)}$ for v_6 . Linear flow generation w_6 is ignored because of its comparably small contributions.

these two limits, with the actual value controlled by the relative magnitudes of the two response coefficients, w_4 and $w_{4(22)}$. When these response coefficients are computed with ideal and viscous hydrodynamics and averaged as in Eq. (3), the event plane correlations seen in Fig. 1(b) are quantitatively reproduced by the response formalism. A more complete comparison of the response formalism to the two and three plane correlations is given in Fig. 2 and Fig. 3.

The non-linear flow response dominates correlation for non-central collisions and for larger values of η/s . Thus, we find that the observed non-trivial event plane correlations become stronger toward non-central collisions as seen in Fig. 2 and Fig. 3. Also as the shear viscosity is increased, the predicted correlations increase and approach the non-linear limit seen in Fig. 1(b). The observed correlation patterns involving Φ_6 have a rich structure. In the response formalism this rich pattern of correlations is due to the fact that two quadratic response coefficients, $w_{6(24)}$ and $w_{6(33)}$, contribute to v_6 .

3. Summary and discussion

Using the PHOBOS MC-Glauber model, $\eta/s = 1/4\pi$, and a freeze-out temperature $T_{fo} = 150$ MeV, the non-linear response formalism qualitatively reproduces the observed two-plane and three-plane correlations, with one exception, $\langle\langle \cos(2\Psi_2 - 6\Psi_3 + 4\Psi_4) \rangle\rangle$. This correlator is currently under investigation. Since we have included only the quadratic response to the lowest deformations, this level of agreement is acceptable. In addition, a preliminary comparison of our predictions to event-by-event hydrodynamics also shows qualitative agreement [8]. In particular, the response formalism reproduces the shear viscosity dependence of the correlations found in event-by-event hydrodynamics. A more complete comparison to event-by-event hydrodynamics and the data is reserved for future work.

Acknowledgements

We would like to thank U. Heinz and Z. Qiu for the helpful discussions. This work is supported in part by the Sloan Foundation and by the Department of Energy through the Outstand Junior Investigator program DE-FG-02-08ER4154.

References

References

- [1] Arsene I *et al.* (BRAHMS Collaboration) 2005 *Nucl.Phys.* **A757** 1–27 (*Preprint nucl-ex/0410020*)
- [2] Adcox K *et al.* (PHENIX Collaboration) 2005 *Nucl.Phys.* **A757** 184–283 (*Preprint nucl-ex/0410003*)
- [3] Muller B, Schukraft J and Wyslouch B 2012 (*Preprint 1202.3233*)
- [4] Gyulassy M and McLerran L 2005 *Nucl.Phys.* **A750** 30–63 (*Preprint nucl-th/0405013*)
- [5] Jia J (ATLAS Collaboration) 2012 (*Preprint 1208.1427*)
- [6] Qiu Z and Heinz U W 2012 *Phys.Lett.* **B717** 261–265 (*Preprint 1208.1200*)
- [7] Gardim F G, Grassi F, Luzum M and Ollitrault J Y 2012 *Phys.Rev.* **C85** 024908 (*Preprint 1111.6538*)
- [8] Qiu Z and Heinz U W 2011 *Phys.Rev.* **C84** 024911 (*Preprint 1104.0650*)
- [9] Teaney D and Yan L 2012 *Phys.Rev.* **C86** 044908 (*Preprint 1206.1905*)
- [10] Teaney D and Yan L 2011 *Phys.Rev.* **C83** 064904 (*Preprint 1010.1876*)